TRINITY COLLEGE

Applications Unit 3 & 4
Test 2 Part A   2016

Calculator Assumed
Sequences

STUDENT'S NAME _____Solution_____

DATE: Thursday 17th March   **TIME:** 30 minutes   **MARKS:** 30

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (2 marks)

Determine the recursive rule for the sequence: $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\ldots$
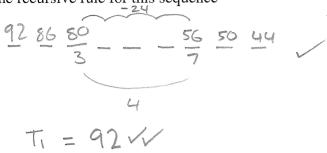
$$T_{n+1} = \frac{1}{2}T_n \checkmark$$

$$T_1 = 1 \checkmark$$

2.   (2 marks)

Determine the first 4 terms of the sequence:

$$T_{n-1} = 2T_{n-2} + 1$$
$$T_1 = 3$$

$3, 7, 15, 31 \ldots$

3.   (5 marks)

An AP has a third term of 80 and a seventh term of 56.

(a)   Determine the recursive rule for this sequence   [4]

$$92 \quad 86 \quad \underset{3}{80} \quad -- \quad -- \quad \underset{7}{56} \quad 50 \quad 44 \quad \checkmark$$

$$T_1 = 92 \checkmark\checkmark$$

$$T_{n-1} = T_n - 6 \checkmark$$

(b)   Determine T_9 [1]

$$T_9 = 44 \checkmark$$

4.   (6 marks)

A tree grows 5 m in its first year. Every subsequent year it grows 30 cm less than the year before until it stops growing altogether.

(a)   Write a recursive rule for the trees growth   [2]

$$T_{n+1} = T_n - 0.3 \checkmark$$
$$T_1 = 5 \checkmark \quad \text{or} \quad \begin{array}{l} T_{n+1} = T_n - 30 \\ T_1 = 500 \end{array}$$

(b)   To the nearest cm how much will the tree grow in its fifth year   [2]

$$\overset{\checkmark \quad \checkmark}{380 \, cm}$$

(c)   How old will the tree be when it stops growing and how tall will it be?   [2]

$$T_{17} = 0.2$$
$$T_{18} = -0.1$$
$$S_{17} = 44.2$$
$$S_{18} = 44.1$$

18 years Old $\checkmark$

44m $\checkmark$

5. (4 marks)

Kate is starting a new fitness program with a goal of running 100 km in the month of June by running every day of the month. On the 1st June, Kate runs 1.7 km. Every day afterwards she increases her run by 4%.

(a) Write a recursive rule for Kate's running [2]

$$T_{n+1} = 1.04 \, T_n \quad \checkmark$$
$$T_1 = 1.7 \quad \checkmark$$

(b) Will Kate achieve her goal? Justify your answer [2]

$$T_{30} = 5.30 \qquad S_{30} = 95.344 \; \checkmark$$

No, she will achieve it on the 1st July. $\checkmark$

6. (5 marks)

Johnny has $2000 to invest at his bank at a rate of 6% per annum. Tom has $1500 to invest at his bank with a rate of 7% per annum. If they both begin their investments at the same time. After how many years will their investment be worth the same amount.

Johnny
$$T_{n+1} = 1.06 \, T_n$$
$$T_0 = 2000 \quad /$$

OR

Investment $= 2000 \times 1.06^n$ $\checkmark$

Tom
$$T_{n+1} = 1.07 \, T_n$$
$$T_0 = 1500 \quad \checkmark$$

Investment $= 1500 \times 1.07^n$ $\checkmark$

$$2000 \times 1.06^n = 1500 \times 1.07^n \; \checkmark$$
$$n = 30.63 \; \checkmark \qquad \text{After 31 years} \; \checkmark$$

OR

$$T_{30} = 11487$$
$$T_{31} = 12176$$

$$T_{30} = 11418 \; \checkmark$$
$$T_{31} = 12218$$

7. (6 marks)

In 2006, a country has an initial population of 1 850 300 people. It has a population rate of 1.7% pa. (population rate is the birth rate take away the death rate) and takes in approximately 80 000 immigrants per year.

(a) Write a recursive rule for the population [2]

$$T_{n+1} = 1.017 T_n + 80\,000$$
$$T_0 = 1\,850\,300$$

(b) Determine the population in 2015 [1]

$$T_9 = 2.9 \times 10^6$$

or

$$2\,924\,382$$

$T_1 = 7$
$T_2 = 8$
$T_3 = 9$
$T_4 = 10$
$T_5 = 11$
$T_6 = 12$
$T_7 = 13$
$T_8 = 14$
$T_9 = 15$

(c) In what year would the population triple the 2006 population. Justify your answer. [3]

$$1\,850\,300 \times 3 = 5\,550\,900$$

$$T_{26} = 5\,456\,506$$

$$T_{27} = 5\,629\,266 \qquad T_{27}$$

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